Predicted Signatures at the LHC from $U(1)$ Extensions of the Standard Model[1].

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Abstract

We discuss the $U(1)_X$ extensions of the standard model with focus on the Stueckelberg mechanism for mass growth for the extra $U(1)_X$ gauge boson. The assumption of an axionic connector field which carries dual $U(1)$ quantum numbers, i.e., quantum numbers for the hypercharge $U(1)_Y$ and for the hidden sector gauge group $U(1)_X$, allows a non-trivial mixing between the mass growth for the neutral gauge vector bosons in the $SU(2)_L \times U(1)_Y$ sector and the mass growth for the vector boson by the Stueckelberg mechanism in the $U(1)_X$ sector. This results in an extra Z' which can be very narrow, but still detectable at the Large Hadron Collider (LHC). The $U(1)_X$ extension of the minimal supersymmetric standard model is also considered and the role of the Fayet-Illiopoulos term in such an extension discussed. The $U(1)_X$ extensions of the SM and of the MSSM lead to new candidates for dark matter.

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Introduction: In lecture I we have discussed high scale models based on supersymmetry, local supersymmetry and supergravity [1, 2]. Specifically we focused on supergravity grand unified models[3] and their experimental consequences, their extensions and tests at the large Hadron Collider (LHC)[4]. Here we discuss extensions of the standard model(SM) and of the minimal supersymmetric standard model(MSSM) to include extra $U(1)$ factors. Such extra $U(1)$ factors appear in a variety of unified models: in grand unified models, i.e., in string models and in D brane models. Thus in $SO(10)$ and E_6 grand unification one has

$$SO(10) \supset SU(5) \times U(1)_\chi; \; E_6 \supset SO(10) \times U(1)_\psi \tag{1}$$

Similarly D brane constructions naturally have many $U(1)$ factors since here on typically starts with a stack of n branes which has a $U(n)$ gauge symmetry. Since $U(n) \supset SU(n) \times U(1)$, one has $U(1)$ factors appearing. Thus, for example, to construct the standard model gauge group in D-brane models one starts with the gauge groups $U(3) \times U(2) \times U(1)^2$ which results in $SU(3) \times SU(2) \times U(1) \times U(1)^3$. Typically the breaking of the extra $U(1)$ symmetry will come about by a Higgs mechanism. Thus if a field has non-vanishing $U(1)$ quantum number, then spontaneous breaking which gives a vacuum expectation value (VEV) to that field will break the extra $U(1)$ symmetry and make the extra gauge boson massive. This is illustrated by the following example: Under $SU(5) \times U(1)_\chi$, the 45 plet of $SO(10)$ has the following decomposition, $45 = 1(0) + 10(4) + \overline{10}(-4) + 24(0)$. The fields which can gain vacuum expectation values without violating charge or color are $1(0)$ and $24(0)$, which however has vanishing $U(1)_\chi$ quantum numbers. Thus while the 45 plet breaks $SO(10)$ it does not reduce the rank and the $U(1)_\chi$ gauge boson remains massless. To reduce the rank one needs a 16 plet or a 126 plet of Higgs. Thus under $SU(5) \times U(1)_\chi$ the 16 plet of Higgs has the decomposition $16 = 1(-5) + \bar{5}(-3) + 10(-1)$, while the 126 plet has the decomposition $126 = 1(-10) + \bar{5}(-2) + 10(-6) + \overline{15}(6) + 45(2) + \overline{50}(-2)$. It is then clear that the VEV growth for 16 or 126 will lead to a rank reduction. Thus a combination of 45 and 16 or a combination of 45 and 126 can reduce $SO(10)$ to the standard model gauge group.

More recently it has been shown that one can accomplish the above with a single irreducible Higgs representation, specifically a 144 plet representation of $SO(10)$[5]. This is so because under $SU(5) \times U(1)_\chi$, the 144 plet decomposes as follows: $144 = \bar{5}(3) + 5(7) + 10(-1) + 15(-1) + 24(-5) + 40(-1) + \overline{45}(3)$. In this case the 24-plet of $SU(5)$ in the above decomposition carries a $U(1)$ quantum number, and thus the $SO(10)$ gauge group breaks directly to the SM gauge group after 144 plet develops a VEV. Further, it may happen that one or more of the extra $U(1)'s$ may remain unbroken at the GUT

scale, and such breaking may only occur at the electroweak scale. In this case the mass of the extra gauge boson which we shall generically call a Z' will have a mass of the electroweak size. Further, if the extra $U(1)$ is coming from a grand unified group the size of the gauge coupling would typically be the size of the gauge couplings in the electroweak group, i.e., typically of size g_2. Thus one generally expects decay widths for the Z' gauge bosons in such scenarios to be size O(GeV).

The outline of the rest of the paper is as follows: We first discuss the Stueckelberg extension of the standard model and the electroweak constraints on it[6, 7] [8, 9]. Next we discuss the Stueckelberg extension of MSSM[10]. This is followed by a discussion of the LHC signatures of Stueckelberg extensions. Of specific interest here is the possibility of a narrow Z'. Finally we discuss the possibility of dark matter arising from the hidden sector of this theory[11, 12] consisitent with the WMAP[13] constraints. In this context we also discuss the Stueckelberg extension to include kinetic energy mixing. Conclusions are given at the end. There are many related works on $U(1)$ extensions (see, e.g., [14], and for a review see [15]) as well as works where hidden sectors play a central role[16, 17, 18, 19, 20, 21, 22, 23, 24]. Further recent works regarding Stueckelberg extensions in the context of the string and D brane models can be found in [25, 26, 27] and in [28, 29].

We begin by introducing the Stueckelberg mechanism[30, 31] for mass growth for $U(1)$ gauge fields. In its simplest form the Stueckelberg mechanism works as follows: consider the Lagrangian given by

$$L = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2}(mA_\mu + \partial_\mu\sigma)^2 + gA_\mu J^\mu \tag{2}$$

where σ is an axionic like field and J^μ is a conserved current. The Lagrangian is invariant under the transformation

$$\delta A_\mu \to \partial_\mu\epsilon, \ \delta\sigma \to -m\epsilon. \tag{3}$$

The Stueckelberg mechanism is endemic in extra dimension models, in strings, and in D branes. Thus consider, for example, the compactification of a 5D theory on S^1/Z_2 (see, e.g., [32]). As an illustration consider the kinetic energy of a $U(1)$ gauge field $A_a(a = 0, 1, 2, 3, 5)$ in 5D,

$$L_5 = -\frac{1}{4}F_{ab}(z)F^{ab}(z), \ a = 0, 1, 2, 3, 5, \tag{4}$$

where $z^a = (x^\mu, y)$, $\mu = 0, 1, 2, 3$ and we may write $A_a = (A_\mu(z), \phi(z))$. After compactification on the fifth co-ordinate y on a half circle, one has the Lagrangian in 4D with

one mass less mode and an infinite number of massive Kaluza-Klein modes

$$L_4 = -\frac{1}{4}\sum_{n=0}^{\infty} F_{\mu\nu}(x)^{(n)} F^{\mu\nu(n)}(x) - \frac{1}{2}\sum_n M_n^2 (A_\mu^{(n)}(x) + \frac{1}{M_n}\partial_\mu \phi^{(n)}(x))^2 + \cdots. \tag{5}$$

In string models and in D brane models the Stueckelberg phenomenon arises naturally from the Green-Schwarz mechanism from a two form field $B_{\mu\nu}$. Thus consider the Lagrangian with a $U(1)$ gauge field A_μ, and a coupling with the two form field $B_{\mu\nu}$ so that [33, 34, 35]

$$L_0 = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{12}H^{\mu\nu\rho}H_{\mu\nu\rho} + \frac{m}{4}\epsilon^{\mu\nu\rho\sigma}F_{\mu\nu}B_{\rho\sigma}, \tag{6}$$

where $H_{\mu\nu\rho}$ is the field strength of the two form field $B_{\mu\nu}$ so that $H_{\mu\nu\rho} = \partial_\mu B_{\nu\rho} + \partial_\nu B_{\rho\mu} + \partial_\rho B_{\mu\nu}$. It is convenient to write the last term as [35] $-\frac{m}{6}\epsilon^{\mu\nu\rho\sigma}(H_{\mu\nu\rho}A_\sigma + \sigma\partial_\mu H_{\nu\rho\sigma})$. An intergration over σ and insertion back gives L_0. Next suppose one solves for $H^{\mu\nu\rho}$ which gives $H^{\mu\nu\rho} = -m\epsilon^{\mu\nu\rho\sigma}(A_\sigma + \partial_\sigma\sigma)$. An integration on $H^{\mu\nu\rho}$ gives L_0 in the form[35]

$$L_1 = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{m^2}{2}(A_\sigma + \partial_\sigma\sigma)^2. \tag{7}$$

Thus we see that the presence of the Green-Schwarz term leads us to mass growth for the $U(1)$ gauge field by the Stueckelberg mechanism.

The Stueckelberg mechanism to give mass to the vector bosons works whether or not the extra $U(1)_X$ is anomalous. Thus in general, an anamolous $U(1)$ gives at one loop[36, 37]

$$\delta L_{1loop} = \lambda c Tr(G \wedge G), \ \delta A_\mu = \partial_\mu\lambda. \tag{8}$$

This term is cancelled by variation of the effective tree level term[36]

$$L_0 = \cdots + mA^\mu\partial_\mu\sigma + \frac{c\sigma}{m}Tr(G \wedge G),$$
$$\delta L_0 = -\lambda c Tr(G \wedge G), \quad \delta\sigma = -\lambda m, \tag{9}$$

so that

$$\delta L_{1loop} + \delta L_0 = 0. \tag{10}$$

Thus one has the following two cases: (i) Anomalous $U(1)$ case, $c \neq 0$, $m \neq 0$: Here one cancels the anomaly and at the same time one has mass growth by the Stueckelberg mechanism for the $U(1)_X$ gauge vector boson; (ii) Non-anomalous case, $c = 0$, $m \neq 0$. In this case there is no need for anomaly cancellation, but there is still mass growth for the vector boson by the Stueckelberg mechanism. Anomalous $U(1)$ in the above context have been discussed by a number of authors (see, e.g.,[38, 26, 39, 40]).

Stueckelberg vs the Higgs mechanism: Next we explore the connection between the Stueckelberg mechanism and the Higgs mechanism[34]. Consider a $U(1)$ gauge theory with a Higgs potential so that

$$L_0 = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - (D^\mu\phi)^\dagger D_\mu\phi - V(\phi) + L_{gf},$$
$$V(\phi) = \mu^2(\phi^\dagger\phi) + \lambda(\phi^\dagger\phi)^2. \tag{11}$$

With $\mu^2 < 0$, and $\lambda > 0$ a spontaneous breaking of the $U(1)$ gauge symmetry occurs, and one has

$$\phi = \frac{1}{\sqrt{2}}(\rho + v)e^{ia/v}, v = \sqrt{-\mu^2/\lambda}. \tag{12}$$

In the limit $(-\mu^2, \lambda) \to \infty$ with $M = ev$ fixed, ρ becomes infinitely heavy and decouples from the rest of the system, and the residual Lagrangian is given by[34]

$$L = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2}M^2(A_\mu - \frac{1}{M}\partial_\mu a)^2 + L_{gf}. \tag{13}$$

Thus we see that in the limit considered above the Higgs mechanism leads to the Stueckelberg mechanism in a very direct way.

Stueckelberg extension of the Standard Model: Although the Stueckelberg mechanism has been around for a long time, a successful incorporation of this mechanism into particle physics was made only recently[6, 10, 36] and its phenomenological implications investigated[7, 8, 9, 41, 12, 42]. Specifically one can extend the standard model by considering the gauge group $SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_X$. We will assume that the visible sector matter fields (quarks, leptons, Higgs) are neutral under the gauge group $U(1)_X$, while the hidden sector fields are neutral under the standard model gauge group. Let the gauge fields for $U(1)_Y$ and $U(1)_X$ be B_μ and C_μ. We assume that there exists a connector sector which carries dual quantum numbers, i.e., quantum numbers of the gauge group $U(1)_Y$ and of $U(1)_X$. Specifically we assume that the axionic field σ is indeed this connector field. Then we append to the SM Lagrangian the following Stueckelberg extension

$$L_{St} = -\frac{1}{4}C_{\mu\nu}C^{\mu\nu} + g_X C_\mu J_X^\mu - \frac{1}{2}(\partial_\mu\sigma + M_1 C_\mu + M_2 B_\mu)^2, \tag{14}$$

where J_X^μ is a conserved current arising from the hidden sector. The above Lagrangian is invariant under the $U(1)$ transformations

$$\delta_Y B_\mu = \partial_\mu\lambda_Y, \quad \delta_Y C_\mu = 0, \quad \delta_Y \sigma = -M_2\lambda_Y, \tag{15}$$

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and under the $U(1)_X$ transformations

$$\delta_X C_\mu = \partial_\mu \lambda_X, \quad \delta_X B_\mu = 0, \quad \delta_X \sigma = -M_1 \lambda_X. \tag{16}$$

The total Lagrangian is of course $L_{SM} + L_{St}$. After spontaneous breaking of the electroweak symmetry, the above Lagrangian gives a mass^2 matrix of the form

$$M_{ab}^2 = \begin{pmatrix} M_1^2 & M_1 M_2 & 0 \\ M_1 M_2 & M_2^2 + \frac{1}{4}g_Y^2 v^2 & -\frac{1}{4}g_Y g_2 v^2 \\ 0 & -\frac{1}{4}g_Y g_2 v^2 & \frac{1}{4}g_2^2 v^2 \end{pmatrix}, \tag{17}$$

where we use a basis $(V_\mu^{\mathrm{T}})_a = (C_\mu, B_\mu, A_\mu^3)_a$ and use the standard form $-\frac{1}{2}V_{a\mu}M_{ab}^2 V_b^\mu$ for the mass term in the Lagrangian. In Eq.(17), g_2 and g_Y are the $SU(2)_L \times U(1)_Y$ gauge coupling constants and $v = 2M_{\mathrm{W}}/g_2 = (\sqrt{2}G_F)^{-\frac{1}{2}}$, where M_{W} is the mass of the W boson, and G_F is the Fermi constant.

The eigen modes of the mass matrix of the vector bosons give a mass less mode which is a photon γ and two massive modes which are Z and Z'. The definition of the electric charge is modified so that

$$\frac{1}{e^2} = \frac{1}{g_2^2} + \frac{1 + \epsilon^2}{g_Y^2}, \tag{18}$$

where $\epsilon = M_2/M_1$. Fits to the precision electroweak data require that ϵ be very small, i.e., $\epsilon \le .06$[8]. The smallness of ϵ leads to the existence of a very narrow resonance if there is no matter in the hidden sector, or if the Z' is forbidden kinematically to decay into the hidden sector matter. This can happen if, for example, $M_{Z'} < 2M_{hid}$ where M_{hid} is the mass of the hidden sector fermion. Further, it also follows that if there is matter in the hidden sector, such matter will couple to the photon with a milli charge strength, i.e., with strength $Q_\epsilon = \epsilon e$. Detailed fits to the LEP I data show that one can satisfy the LEP I constraints to essentially the same level of confidence as fits to the standard model.

LEP II constraints: These arise from the constraints on the contact interaction[43]

$$L_C \sim \frac{g^2 \eta_{sign}}{(1 + \delta)\Lambda^2} \sum_{i,j=L,R} \bar{e}_i \gamma^\mu e_i \bar{f}_j \gamma_\mu f_j. \tag{19}$$

where $\delta = 0$ for $f \ne e$, and $\delta = 1$ for $f = e$, η_{ij} gives the relative contribution of the different chiralities, and η_{sign} tells us if the contribution is constructive or destructive relative to the SM contribution. The most stringent constraints arise from Λ_{VV} for which LEP II gives

$$\Lambda_{VV} > 21.7 \mathrm{GeV}, \tag{20}$$

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while the Stueckelberg extended model gives[9]

$$\Lambda_{VV} = \frac{M_{Z'}}{M_Z} \left(\frac{4\pi}{\sqrt{2} G_F v_e'^2} \right)^{1/2}. \tag{21}$$

For the Stueckelberg extension the LEP II constraints are automatically satisfied once one satisfies the LEP I constraints at the 1% level.

The Stueckelberg extension differs from the previous models in that the photon field is now a linear combination of three gauge fields, A_μ^3, B_μ, C_μ so that

$$A_\mu^\gamma = -c_\theta s_\phi C_\mu + c_\theta c_\phi B_\mu + s_\theta A_\mu^3. \tag{22}$$

This is to be compared with the form of A_μ^γ in the standard model where one has $A_\mu^\gamma = c_\theta B_\mu + s_\theta A_\mu^3$, so it is a combination of only the fields B_μ and A_μ^3. It is precisely because A_μ^γ has a small dependence on the field C_μ that the photon is able to couple to the hidden sector matter with milli charge strength. The model predicts a very narrow Z' resonance in absence of its decay into the hidden sector matter which is in contrast to rather broad resonances arising from Kaluza-Klein excitation of a compact extra dimension[44, 45]. Electroweak tests of the Stueckelberg extension of the standard model can be found in [7, 8].

Stueckelberg extension of the minimal supersymmetric standard model: We consider now the Stueckelberg extension of the minimal supersymmetric standard model(MSSM), and to this end we introduce vector super fields for the $U(1)_Y$ and the $U(1)_X$ gauge groups which we label as B and C respectively. Additionally we introduce the chiral multiplets S and $\bar{S}$ which contain the axionic field and we choose for the Stueckelberg Lagrangian the form

$$\int d\theta^2 d\bar{\theta}^2 (M_1 C + M_2 B + S + \bar{S})^2. \tag{23}$$

The above Lagrangian is invariant under the following $U(1)_Y$ and $U(1)_X$ transformation

$$\delta_Y B = \Lambda_Y + \bar{\Lambda}_Y, \ \delta_Y S = -M_2 \Lambda_Y,$$
$$\delta_X C = \Lambda_X + \bar{\Lambda}_X, \ \delta_X S = -M_1 \Lambda_X. \tag{24}$$

Additionally, of course, we have the gauge kinetic energy terms for gauge multiplet C while the kinetic energy terms for the B multiplet are contained in the MSSM Lagrangian. In the Wess-Zumino gauge the components of the gauge multiplet C are $(C_\mu, \lambda_C, \bar{\lambda}_C, D_C)$, and similarly for the gauge multiplet B, while the chiral multiplet S

has the components $S = (\rho + i\sigma, \chi, F_S)$ and similarly for $\bar{S}$. It is then possible to form two additional Majorana spinors beyond those in MSSM. Here one has

$$\psi_S = \begin{pmatrix} \chi_\alpha \\ \bar{\chi}^{\dot\alpha} \end{pmatrix}, \quad \lambda_X = \begin{pmatrix} \lambda_{C\alpha} \\ \bar{\lambda}^{\dot\alpha}_C \end{pmatrix}. \tag{25}$$

Including the above two, one has six Majorana basis states

$$\psi_S, \lambda_X, \lambda_Y, \lambda_3, \tilde{h}_1, \tilde{h}_2, \tag{26}$$

where $\lambda_Y, \lambda_3, \tilde{h}_1, \tilde{h}_2$ are the four familiar gaugino and higgsino states of MSSM. As in SUGRA models, there is mixing among the six neutral states which gives rise to the six neutralino states in the mass diagonal basis. We can label these states as

$$\xi_1^0, \xi_2^0, \chi_1^0, \chi_2^0, \chi_3^0, \chi_4^0, \tag{27}$$

where χ_i^0 $(i = 1 - 4)$ are the familiar four neutralino states in MSSM, and ξ_1^0 and ξ_2^0 are the two additional neutral states that arise from the Stueckelberg sector. An interesting situation arises when the LSP of the entire system is in the Stueckelberg sector, i.e., it is ξ_1 which we assume is the lighter of $\xi_i (i = 1, 2)$. We will discuss this possibility in the context of dark matter later.

Higgs sector in Stueckelberg extension of MSSM: The Higgs sector is affected in the Stueckelberg extension of MSSM. This is so because one has in the Stueckelberg extension of MSSM the field ρ which couples with the CP even MSSM higgs fields H^0, h^0 giving a 3×3 mass matrix. We display this mass matrix below[36]

$$(M_{H^0}^2) = \begin{pmatrix} (M_Z^2 c_\beta^2 + m_A^2 s_\beta^2) & -(M_Z^2 + m_A^2)s_\beta c_\beta & -s_\theta c_\beta M_Z M_2 \\ -(M_Z^2 + m_A^2)s_\beta c_\beta & (M_Z^2 s_\beta^2 + m_A^2 c_\beta^2) & s_\theta s_\beta M_Z M_2 \\ -s_\theta c_\beta M_Z M_2 & s_\theta s_\beta M_Z M_2 & m_\rho^2 \end{pmatrix}, \tag{28}$$

where $s_\beta(c_\beta) = \sin\beta(\cos\beta)$, $s_\theta = \sin\theta_W$ where θ_W is the weak angle. The above matrix in the neutral Higgs sector has the eigen states H_1^0, H_2^0, H_3^0. One may choose these so that $H_1^0, H_2^0, H_3^0 \to H^0, h^0, \rho$ for the case when the coupling to the Stueckelberg sector vanishes, i.e., $M_2 = 0$, where h^0 is the light neutral Higgs of MSSM and H^0 is the heavy neutral Higgs of MSSM.

The Stueckelberg mechanism and the Higgs mechanism in MSSM: In the non-supersymmetric case it was shown that the Higgs mechanism reduces to the Stueckelberg mechanism[30] in an appropriate limit with a constraint on the mass[2] and coupling constant parameters of the Higgs potential. In the supersymmetric case, it turns out that one needs

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Fayet-Iliopoulos D-terms to accomplish this reduction[41]. Thus we begin by adding Fayet-Iliopoulos D terms to the scalar potential

$$L_{FI} = \xi_X D_C + \xi_Y D_Y \tag{29}$$

Including these the D-part of the scalar potential has the form

$$V_D = \frac{g_X^2}{2}(Q_X|\phi^+|^2 - Q_X|\phi^-|^2 + \xi_X)^2 + \frac{g_Y^2}{2}(Y_\phi|\phi^+|^2 - Y_\phi|\phi^-|^2 + \xi_Y)^2 \tag{30}$$

Minimization of the potential gives $\langle\phi^+\rangle = 0$, $\langle\phi^-\rangle \neq 0$ and

$$M_1 = \sqrt{2}g_X Q_X\langle\phi^-\rangle, M_2 = \sqrt{2}g_Y Y_\phi\langle\phi^-\rangle, \tag{31}$$

The limit $g_X Q_X \to 0, g_Y Y_\phi \to 0$ with M_1, M_2 fixed reduces the Higgs mechanism above to the Stueckelberg form[41].

Kinetic mixing: The kinetic energy of two $U(1)'s$ will mix if there are fields even with GUT size mass which carry dual quantum numbers[16]. Thus consider two $U(1)$ gauge fields A_1^μ, A_2^μ with the interaction

$$L_{km} = -\frac{1}{4}F_{1\mu\nu}F_1^{\mu\nu} - \frac{1}{4}F_{2\mu\nu}F_2^{\mu\nu} - \frac{\delta}{2}F_{1\mu\nu}F_2^{\mu\nu} + J'_\mu A_1^\mu + J_\mu A_2^\mu. \tag{32}$$

where J_μ is the source arising from the visible sector matter fields and J'_μ is the source containing fields in the hidden sector. We can go to the diagonal basis with the transformation

$$\begin{pmatrix} A_1^\mu \\ A_2^\mu \end{pmatrix} \to K_0 \begin{pmatrix} A'^\mu \\ A^\mu \end{pmatrix}, \quad K_0 = \begin{pmatrix} \frac{1}{\sqrt{1-\delta^2}} & 0 \\ \frac{-\delta}{\sqrt{1-\delta^2}} & 1 \end{pmatrix}. \tag{33}$$

However, there is a degree of arbitrariness in the choice of the matrix that diagonalizes the kinetic energy matrix. Specifically one may take K instead of K_0 as the diagonalizing matrix where

$$K = K_0 R, \quad R = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}. \tag{34}$$

A convenient choice of θ is

$$\theta = arctan[\delta/\sqrt{1-\delta^2}] \tag{35}$$

which gives the asymmetric solution

$$L_1^K = A^\mu \left[\frac{1}{\sqrt{1-\delta^2}}J_\mu - \frac{\delta}{\sqrt{1-\delta^2}}J'_\mu \right] + A^{\mu'}J'_\mu \tag{36}$$

We identify A^μ with the photon field which has interactions with the visible sector source J_μ with normal strength and also an interaction with the hidden sector source J'_μ with a strength which is proportional to δ and thus this interaction is milli charge size. The field A'_μ is another massless vector field which interacts with the hidden sector source J'_μ and has no interactions with the visible sector current J_μ.

Next let us consider the Stueckelberg mechanism with kinetic energy mixing. Here in addition to the kinetic mixing one also has mass mixings so that

$$L_{St}^m = -\frac{1}{2}M_1^2 A_{1\mu}A_1^\mu - \frac{1}{2}M_2^2 A_{2\mu}A_2^\mu - M_1 M_2 A_{1\mu}A_2^\mu. \tag{37}$$

Diagonalization of the mass matrix fixes θ so that

$$\theta = arctan\left(\frac{\epsilon\sqrt{1-\delta^2}}{1-\epsilon\delta}\right). \tag{38}$$

With the above one finds the following interactions in the basis where both the mass and the kinetic energy terms are diagonal

$$L_{St}^{int} = \frac{1}{\sqrt{1-2\epsilon\delta+\epsilon^2}}\left(\frac{\epsilon-\delta}{\sqrt{1-\delta^2}}J_\mu + \frac{1-\epsilon\delta}{\sqrt{1-\delta^2}}J'_\mu\right)A_M^\mu$$
$$+ \frac{1}{\sqrt{1-2\epsilon\delta+\epsilon^2}}\left(J_\mu - \epsilon J'_\mu\right)A_\gamma^\mu. \tag{39}$$

Here A_γ^μ is the photonic field and A_M^μ is the massive vector boson field. We note that the coupling of the photon with the hidden sector is controlled by ϵ and vanishes when ϵ vanishes. This is in contrast to the case of the pure kinetic mixing where the coupling of the photon with the hidden sector is controlled by δ. One can carry out a similar extension of the Stueckelberg extension of the standard modelgauge group to $SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_X$ with both kinetic mixing and mass mixing for the two $U(1)'s$. In this case one finds that the electric charge is modified so that

$$\frac{1}{e^2} = \frac{1}{g_2^2} + \frac{1-2\epsilon\delta+\epsilon^2}{g_Y^2}. \tag{40}$$

An interesting result arises in that in the absence of the hidden sector, the weak sector of the model depends only on the combination $\bar\epsilon = (\epsilon-\delta)/\sqrt{1-\delta^2}$. In this case the precision LEP data constrains $\bar\epsilon$[12]. For a discussion of other approaches to $U(1)$ probes of the hidden sector see [19, 20, 28].

Milli-weak and hidden sector dark matter in Stueckelberg extensions: The Stueckelberg extension gives rise to two new candidates for dark matter. One of these is milli

weak or extra weak dark matter arising from the Stueckelberg extension of MSSM. The other is dark matter which arises from the hidden sector to which $U(1)_X$ couples. We discuss each of these cases in some detail below.

Milli weak dark matter: In the Stueckelberg extension of MSSM one finds 6 neutralino states four of which are the usual states from the MSSM sector while the remaining two arise from the Stueckelberg sector. Here there are two distinct possibilities[10]. The first one corresponds to the case when χ_1^0 is the LSP. In this case the analysis of dark matter and of the supersymmetric signatures would exactly be the same as for the case of the SUGRA models except for some minor corrections arising from the mixings of the Stueckelberg and the MSSM sector. The second possibility corresponds to the case when ξ_1^0 is the LSP. In this case the LSP is milli weak or extra weakly interacting. The LSP of the MSSM sector would decay into ξ_1^0 inside the detector, and thus one still has missing energy signals. Naively one might expect that because of the extra weak interactions of ξ_1^0 it might be difficult to achieve an efficient annihilation of the ξ_1^0's. However, this is not the case when one takes into account the coannihilations. It is then possible to satisfy the relic density constraints consistent with WMAP data[41, 46]. For other possible candidates arising from $U(1)$ extensions see[47, 48].

Hidden sector dark matter[11, 12]: Hidden sector fermions could be candidates for dark matter. Specifically let us assume that on has massive Dirac fermions with mass m_D in the hidden sector. These would be milli charged (for other recent works with milli charged particles see [49]

Detailed analyses then show that one can satisfy the WMAP relic density constraints[11, 12]. There are two clear regimes for the relic density constraints to be satisfied. The first one is when $M_{Z'} > 2m_D$ in which case the Z' width will be normal size because Z' can decay with normal strength into the Dirac fermions of the hidden sector. However, the Z' coupling with the SM quarks and leptons is small because it is suppressed by a factor ϵ^2. Consequently in this scenario it is difficult to detect the Z' by Drell-Yan at the Large Hadron Collider. The second possibility corresponds to the case when $M_{Z'} < 2m_D$. Here it is still possible to satisfy the relic density constraints by a proper thermal averaging over the Z' pole. However, in this case the Z' cannot decay on shell into the Dirac fermions of the hidden sector but it can do so into the quarks and leptons of SM. Thus one can detect the Z' by the Drell -Yan at the Large Hadron Collider. The Dirac fermions in the hidden sector can explain the positron excess seen by the

anti-matter satellite probe PAMELA[50]. An analysis shows that such an excess can be understood from the annihilation of Dirac fermions in the hidden sector[51]. An important issue emphasized in the analysis of [51] is the enhancement or boost of the ratio $\langle \sigma v \rangle_{Halo}/\langle \sigma v \rangle_{Freezeout}$ near a pole due to the fact that the temperature in the halo is much smaller compared to the temperature at the freezeout. The above phenomenon come about when twice the mass of the annihilating particle is close to the resonance mass in the channel in which they annihilate. For the annihilation of the Dirac particles considered in [51] the resonance is the Stueckelberg Z' pole. In this case the ratio is very sensitive to the temperature, and the velocity averaging in the halo gives larger results due to a much smaller temperature in the halo relative to the velocity averaging at the freezeout where the temperature is much larger $T \sim m_D/(20 - 30)$. Because of the above enhancement much smaller additional boost factors are needed to fit the positron data[51]. For other works on the Stueckelberg mechanism and applications see [52, 53, 21]. and for other works on dark matter from hidden sector see[49, 54].

Conclusion: Extra $U(1)$'s arise in a wide variety of GUT, string and D brane models, and the Stueckelberg extensions of SM provide a natural framework to incorporate them. The Stueckelberg mechanism is an alternative to the Higgs mechanism for the breaking of the $U(1)$ gauge symmetry. It has the advantage over the Higgs mechanism in that one does not need to construct a scalar potential or generate spontaneous breaking to give mass to the gauge vector boson. The Stueckelberg extension leads to new phenomena testable at colliders, specfically a sharp Z' resonance with a width in the MeV to sub GeV range in the absence of decay of the Z' into hidden sector matter. Two new types of dark matter emerge in the Stueckelberg extension. The first is a milli weak dark matter candidate whose interactions with matter are weaker than the weak interactions of the weakly interacting massive particles (WIMPS). The second new candidate for dark matter that emerges is the milli charged dark matter which arises solely from the hidden sector. It is found that both the milli weak and the milli charged dark matter candidates can generate the right amount of dark matter in the universe consistent with the WMAP relic density constraints. Since the Stueckelberg mechanism arises quite naturally in string and D brane models, a confirmation the Stueckelberg mechanism experimentally would lend support to the existence of a new regime of physics - perhaps string theory.

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